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On control of nonlinear system dynamics at unstable steady state

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Abstract

Shifting an oscillatory or a chaotic trajectory to the unstable steady state of a nonlinear system in the presence of stochastic or deterministic load disturbances continues to be a nontrivial task. In the present work, two effective strategies for such control needs are presented. The control laws employed do not contain the process model parameters explicitly. The suggested strategies are demonstrated on two simulated nonlinear reaction systems exhibiting multi-stationarity, limit cycle oscillations, and chaos. © 1997 Elsevier Science S.A.

Keywords: Nonlinear dynamics; Chaos; Control

1. List of symbols

a	Model parameter
b	Model parameter
B	Heat of reaction parameter
c	Model parameter
d_1	Load disturbance in x_1 and x
d_2	Load disturbance in x_2 and y
d_3	Load disturbance in x_3 and z
$\hat{d}$	Model parameter
Da	Damkohler number
e	Set point error
g	Model parameter
$\hat{f}$	Model parameter
K_c	Controller gain
l	Model parameter
$\hat{s}$	Model parameter
S	Ratio of the rate constants for the series reaction (Case study 1)
t	Time
u_t	Manipulated variable
x	Model variable in Case study 2
x_1	Dimensionless concentration of species A
x_1^{set}	Set point for variable x_1
x_2	Dimensionless concentration of species B
x_2^{set}	Set point for variable x_2
x_3	Dimensionless temperature
x_3^{set}	Set point for variable x_3
x_{1s}, x_{2s}, x_{3s}	Steady state values of x_1, x_2 and x_3
x_s	Steady state value of x
y	Model variable in Case study 2
y_s	Steady state value of y
z	Model variable in Case study 2
z_s	Steady state value of z
u_t	Control variable
$\overline{u_t}$	Controller output when setpoint error equal to zero

Greek symbols

α	Ratio of heat effects for the series reaction
β	Heat transfer coefficient
ϵ	Controller gain
ϵ_O	Initial controller gain
κ	Ratio of the activation energies for the series reaction
τ_D	Time constant for derivative action
τ_I	Time constant for integral action

2. Introduction

Control of nonlinear dynamical systems has attracted attention in recent years. In general, control of nonlinear systems is difficult, and this is especially so for systems exhibiting chaotic dynamic behavior. One approach to chaos control,

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proposed by Ott, Grebori and Yorke (OGY) [1], stabilizes the unstable periodic orbits (UPOs) of the system by applying small perturbations in the neighborhood of the desired UPO. Several independent control strategies and modified forms of the OGY methodology have been proposed since then, and used to stabilize UPOs and to suppress chaotic dynamics (see, for example, Hubler [2], Hunt [3], Shinbrot et al. [4–6], Mehta and Henderson [7], Dressler and Nitsche [8], Pyragas [9], Qu et al. [10], Bielawski et al. [11], Paskota et al. [12], Chen and Dong [13]).

The second approach to controlling the unstable behavior of nonlinear systems aims at stabilizing the dynamic trajectory exactly at the unstable steady state (USS). Since a USS repels trajectories in its neighborhood, deriving a control algorithm which ensures that the trajectory stays at an USS is not trivial. In recent years, Singer et al. [14] have demonstrated experimentally and theoretically that the chaotic trajectories can be steadied in a narrow region by using a simple on-off control strategy. The model-based parametric adaptive control strategy proposed by Vassiliadis [15] may also be used to stabilize the chaotic trajectories at an USS. However, this technique requires a phenomenological or empirical process model which in most instances is difficult to formulate. The objective of this work is to design and demonstrate alternative strategies that do not contain model parameters explicitly in the control law for regulating continuous-time nonlinear systems exactly at an unstable steady-state. Towards this goal, two control strategies are proposed and successfully employed to control two well-known simulated nonlinear reaction systems showing unstable behavior such as multi-stationarity, oscillations and chaos.

3. Control strategies

Regardless of the type of application under scrutiny, a control law must exploit the difference between the desired process value, i.e., the set point, and the actual process value to drive the latter towards the set point. Some well-known control laws that so exploit this difference include the PID (proportional-integral-derivative) controller and IMC (internal model control), among others.

A specific expression that has been found to be useful in carrying out controller adjustments in nonlinear systems is of the form

$$\frac{du_t}{dt} = \epsilon(x^{set} - x) \tag{1}$$

where u_t, ϵ and t respectively denote the control parameter, controller gain (tuning parameter) and time. The setpoint for process variable x is represented by x^{set} and the bracketed terms represent the setpoint error e signifying the difference between the target state and the actual process output. Eq. (1) with constant ϵ has been used earlier as an adaptive controller to adjust a system parameter [16,17]. Other studies employ Eq. (1) to control the unstable behaviour as a parameter-adapting mechanism in the framework of model-based strategies like Internal Model Control (IMC) [18,19]. However, in these studies, process model parameters appear explicitly in the control law and they do not address the problem of controlling nonlinear systems exhibiting chaotic motion at an unstable steady-state.

For nonlinear systems exhibiting sustained oscillatory or chaotic behavior with reference to the unstable steady state, the system error switches sign continuously. Thus to control a nonlinear system possessing these characteristics at the unstable steady state, Eq. (1) must be modified. Based on heuristic reasoning, the suggested modification is

$$u_t = \epsilon e \tag{2a}$$

or

$$\frac{du_t}{dt} = \frac{d(\epsilon e)}{dt} \tag{2b}$$

where ϵ is now a time-varying proportionality factor. Eq. (2) can be simplified as

$$\frac{du_t}{dt} = e\frac{d\epsilon}{dt} + \epsilon(t)\left(\frac{de}{dt}\right) \tag{2c}$$

For separable systems

$$\epsilon = \epsilon_0 f(t) \tag{3}$$

and, therefore, Eq. (2c) may be written as

$$\frac{du_t}{dt} = \epsilon_0 f'(t) e + \epsilon_0 f(t)\left(\frac{de}{dt}\right) \tag{4}$$

where prime denotes the time derivative.

As may be seen, the right-hand side (rhs) of this controller equation contains terms which are proportional to the setpoint error e as well as its time derivative. The equation is nonautonomous in character since the gain terms ($\epsilon_0 f'(t)$ and $\epsilon_0 f(t)$) are continuously adapted in relation with time. For the simple choice of $f(t) = t$, and correspondingly $f'(t) = 1$, the gain, $\epsilon_0 f'(t)$, becomes independent of time while the gain ($\epsilon_0 f(t)$) for the derivative action retains its time dependence. Eq. (4) can be viewed as a linear proportional controller where the gain is a function of time. The setpoint error can be defined either as

$$e = (x^{set} - x) \tag{5}$$

or as

$$e = \sum_{i=1}^{n} (x_i^{set} - x_i) \tag{6}$$

where n denotes the number of system variables. It is well known that incorporating integral action improves control performance and eliminates offset. We therefore supplement the control action of Eq. (4) by adding an integral term according to

$$\frac{du_t}{dt} = \epsilon_0 f'(t) e + \epsilon_0 f(t)\frac{de}{dt} + \epsilon_1 \int e \, dt \tag{7}$$

Table 1
Characterization of steady states [a] for systems in case studies 1 and 2

Set. No.	Parameter values	$x_{1,s}$	$x_{2,s}$	$x_{3,s}$	Stability
Case Study 1					
I	$Da=.06$, $S=.0005$, $\epsilon=0$, $\kappa=1$,	0.8965	0.1034	0.6540	Stable
	$\alpha=0.426$, $\beta=7.7$, $B=55.0$	0.6595	0.3404	2.1523	Unstable
		0.0378	0.9501	6.0500	Unstable
II	$Da=0.26$, $S=0.5$, $\epsilon=0$, $\kappa=1$,	0.0729	0.1259	3.890	Stable limit cycle
	$\alpha=0.426$, $\beta=7.7$, $B=57.77$				
III	Same as Set II except	0.0819	0.1391	3.7627	Chaotic
	$\beta=7.9999$				
Case Study 2		x_s	y_s	z_s	
I	$a=0.5$, $\hat{b}=3$, $c=5$, $\hat{\epsilon}=0.01$,	1.24855	1.8705	0.3015	Unstable
	$\hat{f}=0.5$, $g=0.6$, $\hat{s}=0.3$, $\hat{d}=0.51$,	2.2462	0	1.7455	Unstable
	$l=1.339$	7.7463	0	4.5506	Unstable

[a] Denoted by subscript s.

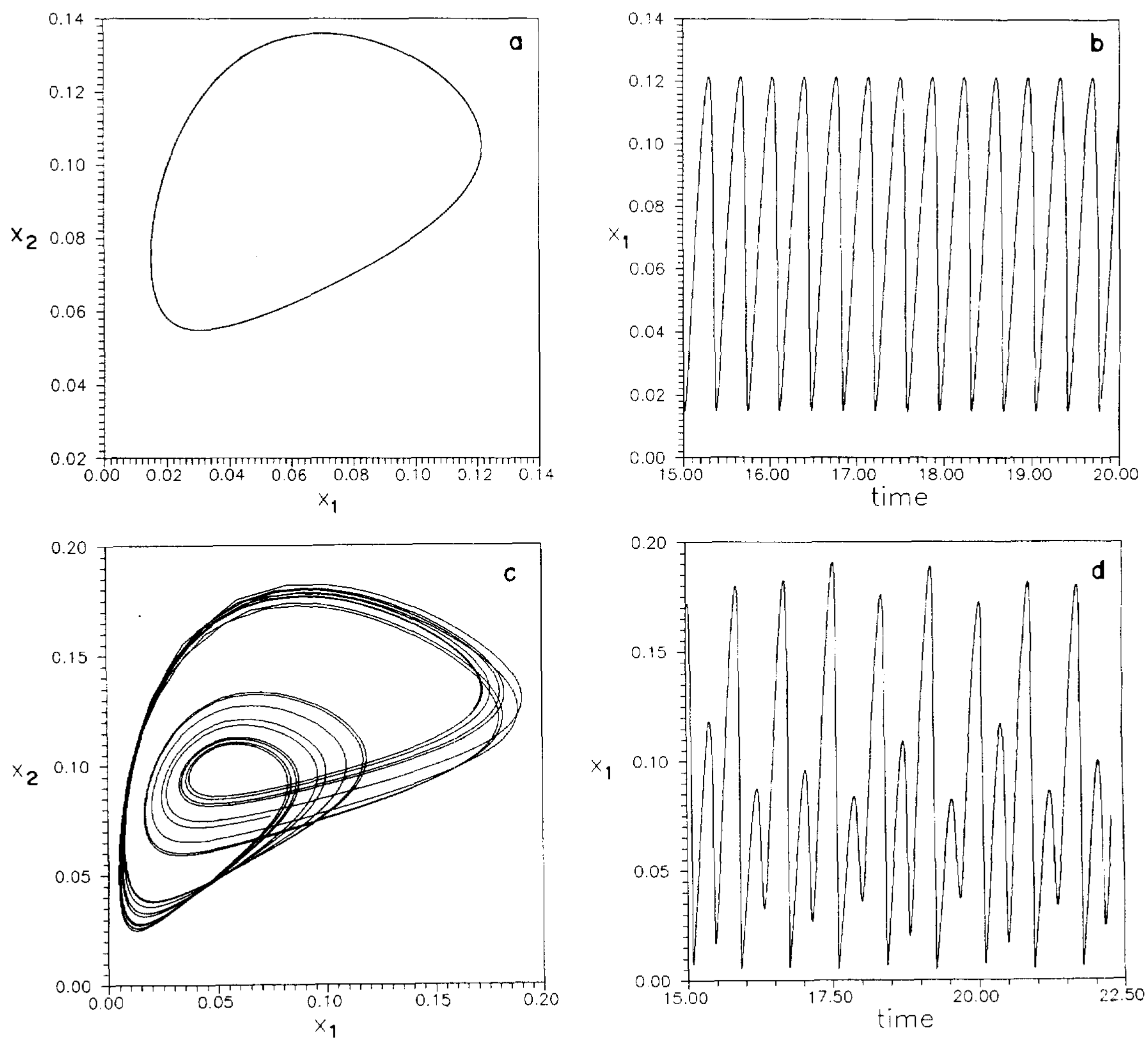

Fig. 1. Plots showing the oscillatory (a, b) and chaotic (c, d) dynamics possessed by the nonisothermal CSTR system for parameters defined by set (2) and (3) respectively. (a) and (c) depict the phase plane plots, while (b) and (d) show the corresponding x_1 profiles in time.

In essence, the control law given by Eq. (7) corresponds to the controller Eq. (4) augmented with a double integrator. The two controllers defined by Eqs. (4) and (7) are the final controller expressions and they will henceforth be referred to as controller-1 and controller-2, respectively. Note that these controllers do not contain the process model parameters

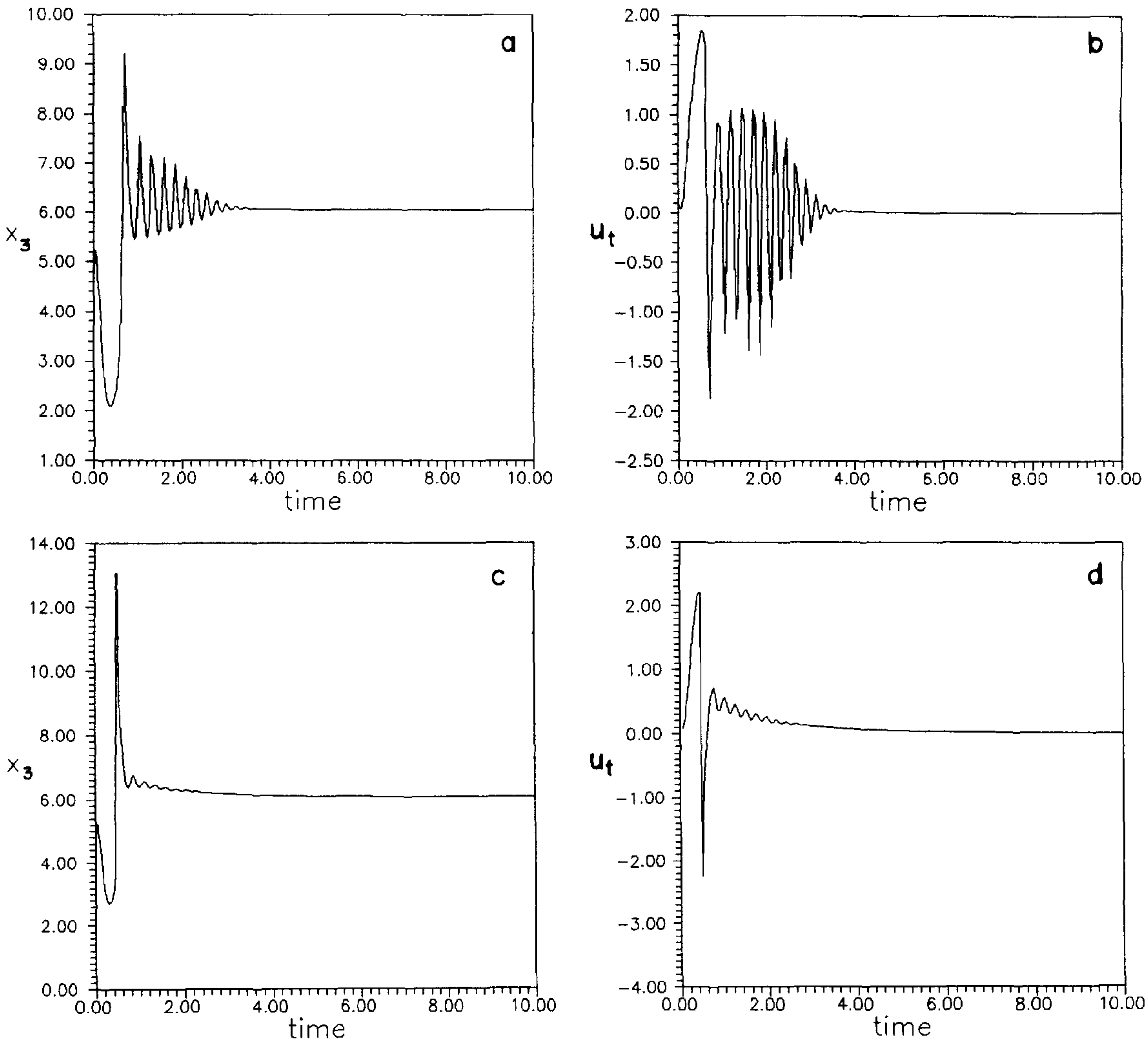

Fig. 2. Plots of process outputs (a, c) and controller outputs (b, d) obtained with controller-1 (a, b) and controller-2 (c, d). The plots pertain to the control objective 1 where the setpoint is a USS belonging to the multi-stationary region (see Table 1, parameter set 1). (a) and (b) show the time profiles of process variable x_3 and controller-1 output u_t. (c) and (d) depict the time profiles of the same variables but for controller-2.

explicitly. In the following sections, the performance of these controllers is tested on two simulated reaction systems governed by a set of coupled nonlinear ordinary differential equations.

4. Case study 1

Consider the reactor model describing the dynamics of an exo-, endothermic reaction in a jacketed continuous stirred tank reactor (CSTR). The reaction is assumed to be first-order irreversible and consecutive of the type $A \rightarrow B \rightarrow C$ and has been studied in detail by Kahlert et al. [20]. The reactor model exhibits diverse dynamic features such as multi-stationarity, limit cycle oscillations and even chaos for certain parameter values. The corresponding dimensionless mass (for species A and B) and energy balance equations are

$$\frac{dx_1}{dt} = 1 - x_1 - Dax_1\exp[x_3/(1+\epsilon_A x_3)] \tag{8}$$

$$\frac{dx_2}{dt} = -x_2 + Dax_1\exp[x_3/(1+\epsilon_A x_3)] - DaSx_2\exp[\kappa x_3/(1+\epsilon_A x_3)] \tag{9}$$

$$\frac{dx_3}{dt} = -x_3 + BDax_1\exp[x_3/(1+\epsilon_A x_3)] - DaB\alpha Sx_2\exp[\kappa x_3/(1+\epsilon_A x_3)] - \beta(x_3 - x_{3c}) \tag{10}$$

Here, x_1 and x_2 represent the dimensionless concentrations of species A and B, respectively, while x_3 denotes the dimensionless CSTR temperature. The definitions of other parameters can be found in Kahlert et al. [20]. The parameter x_{3c} represents the reactor coolant temperature. For control purposes, we define the manipulated control variable, u_t, as the deviation from the reference value of x_{3c}. Consequently, the

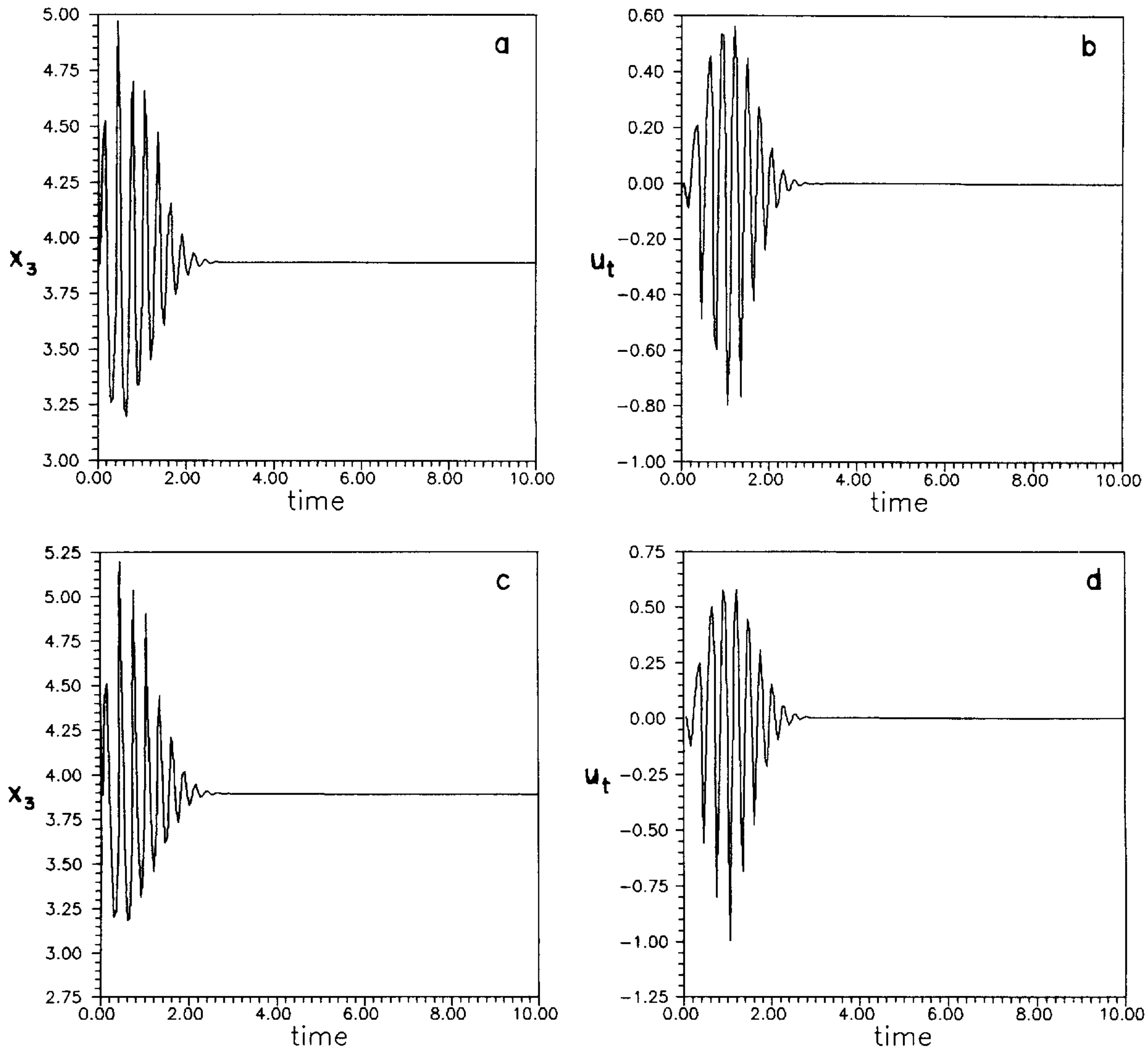

Fig. 3. Plots of process outputs (a, c) and controller outputs (b, d) obtained with controller-1 (a, b) and controller-2 (c, d). The plots pertain to objective 2 where the setpoint is the unique USS exhibiting limit cycle behavior. (a) and (b) show the time profiles of process variable x_3 and controller-1 output u_t. (c) and (d) depict the time profiles of the same variables but for controller-2.

model equations in the presence of controller action and load disturbances become

$$\frac{dx_1}{dt} = 1 - x_1 - Dax_1 \exp[x_3/(1+\epsilon_A x_3)] + d_1 \quad (11)$$

$$\frac{dx_2}{dt} = -x_2 + Dax_1 \exp[x_3/(1+\epsilon_A x_3)] - DaSx_2 \exp[\kappa x_3/(1+\epsilon_A x_3)] + d_2 \quad (12)$$

$$\frac{dx_3}{dt} = -x_3 + BDax_1 \exp[x_3/(1+\epsilon_A x_3)] - DaB\alpha Sx_2 \exp[\kappa x_3/(1+\epsilon_A x_3)] - \beta(x_3 - x_{3c}) + \beta u_t + d_3 \quad (13)$$

where the load disturbances in feed compositions are denoted by d_1 and d_2 and that in temperature is represented by d_3.

To fix up the controller goal, we can utilize the steady state and linear stability analysis of the model equations Eqs. (8)–(10). Such an analysis has been performed by Bandyopadhyay [21] and the parameter values for which the system shows multi-stationarity, oscillations, and chaos are listed in Table 1. For parameter set I, the system admits three steady states of which two are unstable. The parameter sets II and III correspond to unique USSs for which the system exhibits limit cycle oscillations and chaotic behavior, respectively. To test the performance of the proposed controllers, four representative control objectives pertinent to the operation of CSTR have been identified.

The numerical integration of system and controller equations has been performed using Gear's routine with the sampling interval of 0.0001 time units. In both the case studies, the term $f(t)$ appearing in the definition of the time varying proportionality factor ϵ ($=\epsilon_0 f(t)$) is set as $f(t)=t$ (resulting in $f'(t)=1$). Also, the control parameters ϵ_0 and ϵ_1 were set equal to unity. In general, the controller parameters must be found by trial and error but experience suggests that the sys-

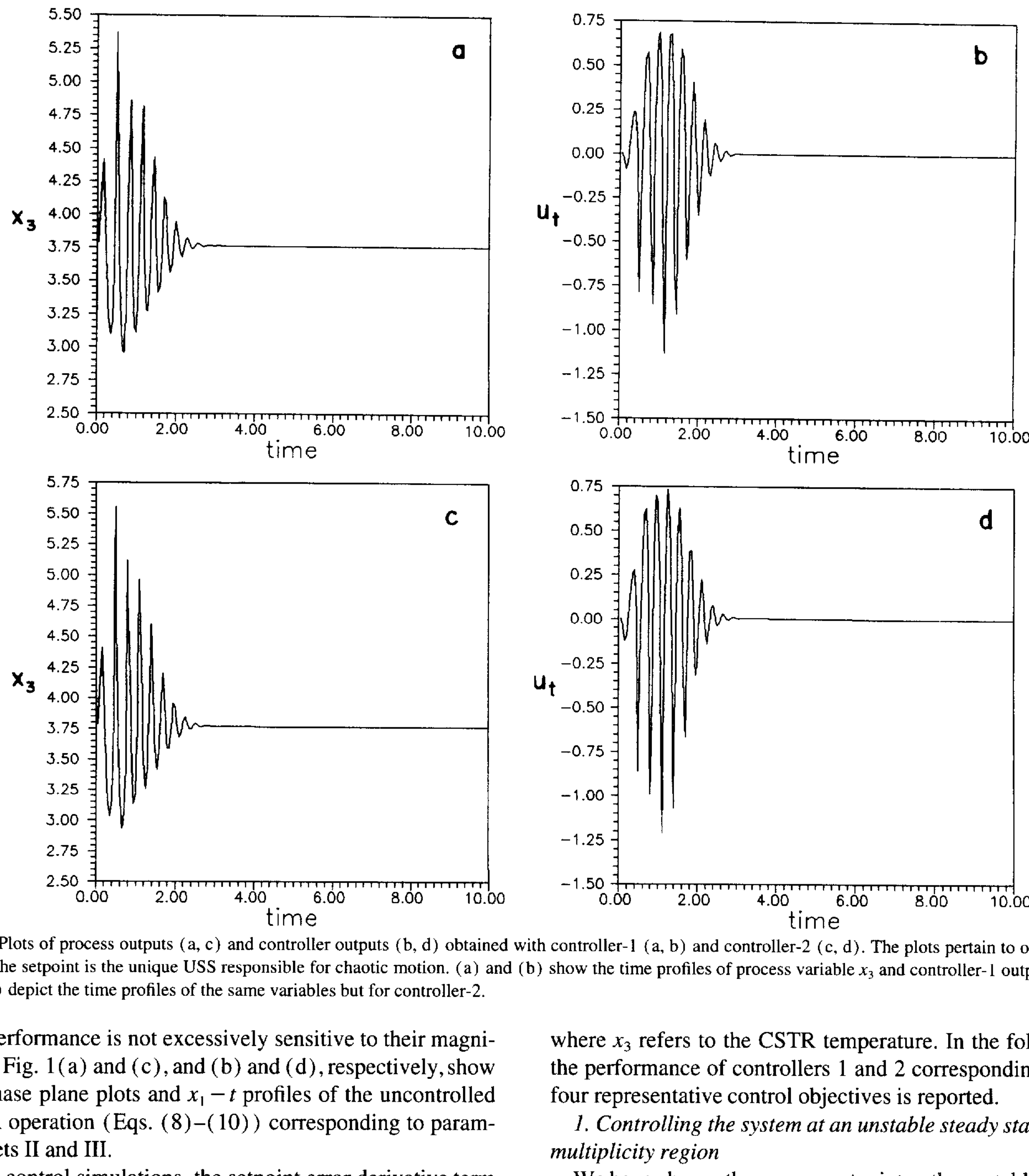

Fig. 4. Plots of process outputs (a, c) and controller outputs (b, d) obtained with controller-1 (a, b) and controller-2 (c, d). The plots pertain to objective 3 where the setpoint is the unique USS responsible for chaotic motion. (a) and (b) show the time profiles of process variable x_3 and controller-1 output u_t. (c) and (d) depict the time profiles of the same variables but for controller-2.

tem performance is not excessively sensitive to their magnitudes. Fig. 1(a) and (c), and (b) and (d), respectively, show the phase plane plots and $x_1 - t$ profiles of the uncontrolled CSTR operation (Eqs. (8)–(10)) corresponding to parameter sets II and III.

For control simulations, the setpoint error derivative term ($\mathrm{d}e/\mathrm{d}t$) was evaluated using the four-point backward finite-difference scheme, and the set point error e was evaluated as

$$e = (x_1^{\text{set}} - x_1) + (x_2^{\text{set}} - x_2) + (x_3^{\text{set}} - x_3) \tag{14}$$

Evaluation of the set point error in this way is possible only when the steady state values of the system variables x_1 and x_2 corresponding to the set point (x_3^{set}) of the controlled variable are known a priori. In the absence of a process model, these steady state values are not known in advance and, therefore, the set point error is evaluated in accordance with Eq. (5) as

$$e = (x_3^{\text{set}} - x_3) \tag{15}$$

where x_3 refers to the CSTR temperature. In the following, the performance of controllers 1 and 2 corresponding to the four representative control objectives is reported.

1. Controlling the system at an unstable steady state in the multiplicity region

We have chosen the process setpoint as the unstable steady state in the multiplicity region ($x_1^{\text{set}} = 0.0378$, $x_2^{\text{set}} = 0.9501$ and $x_3^{\text{set}} = 6.05$). The controller goal is to shift the process operating at an arbitrary point ($x_{10} = 0.04$, $x_{20} = 0.9$, $x_{30} = 5.75$) to the setpoint and maintain it at that state. Fig. 2 shows the plots due to control actions of controller-1 and controller-2, wherein Fig. 2(a) and (c) show the respective $x_3 - t$ profiles. The corresponding controller outputs are plotted in Fig. 2(b) (controller-1) and (d) (controller-2). Note that the control was activated right from time $t = 0$. It can be seen from these figures that both the controllers are capable of shifting the process from an arbitrary point to the target state and also sustaining it at that point. Looking at the respective controlled trajectories, it is also observed that the action

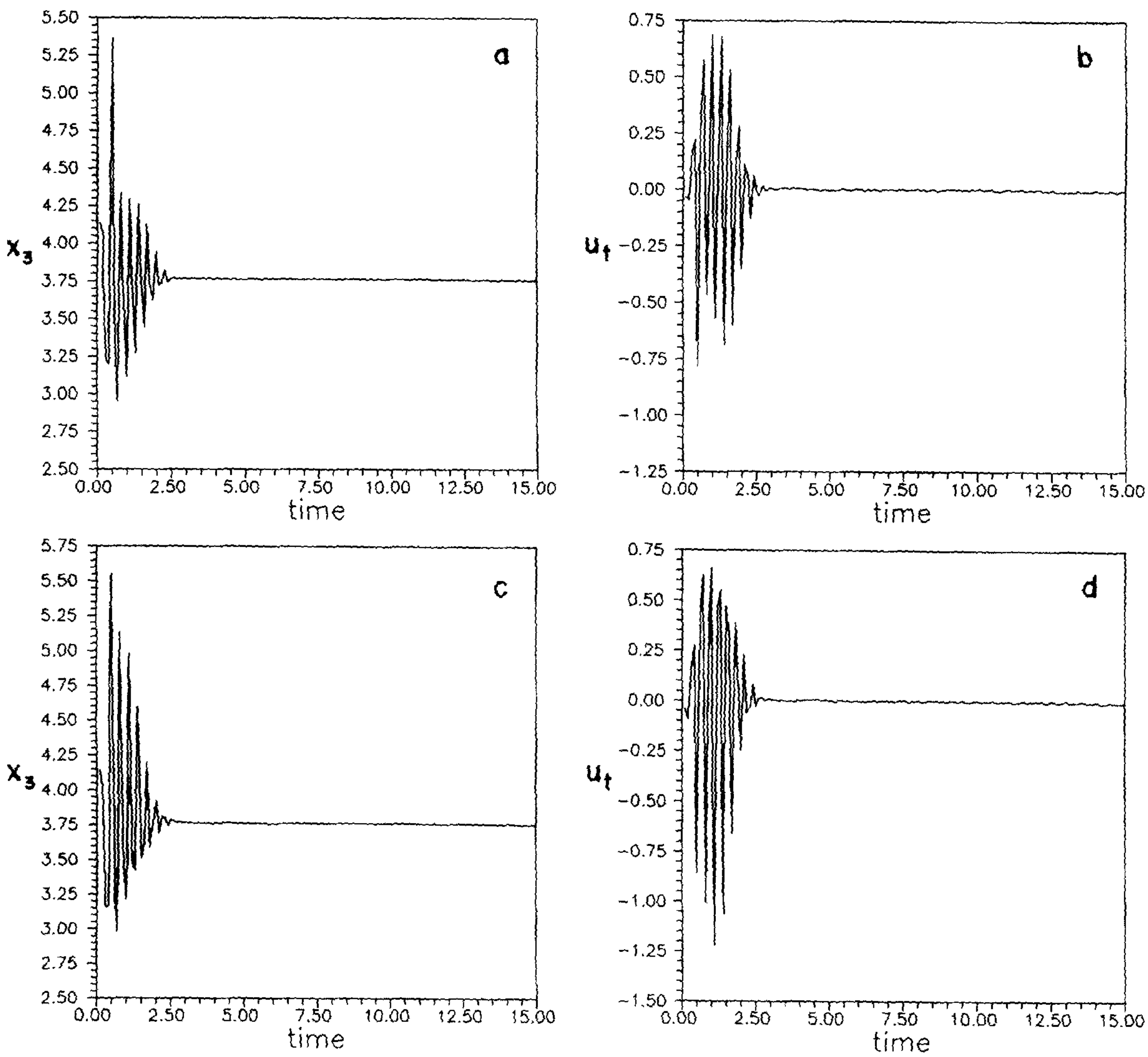

Fig. 5. The description of trajectories is identical to Fig. 4 except that the process now contains stochastic load disturbance (d_3) obeying Gaussian distribution with mean and standard deviation equal to 0 and 0.25 respectively.

of controller-2 is smoother (albeit slower) as compared to that exerted by controller-1.

2. Controlling the system at the unique unstable steady state responsible for limit cycle oscillations

For this case, the model parameters corresponding to the parameter set II for which the system exhibits sustained oscillatory behavior (stable limit cycle) were selected. The controller is first required to shift the process trajectory from an arbitrarily selected initial state; $x_{10}=0.08$, $x_{20}=0.103$ and $x_{30}=3.654$ to the target state; $x_s=0.0729$, $x_{2s}=0.1259$, $x_{3s}=3.89$. The controllers in the same operation are also required to regulate the trajectory at the setpoint representing the unique USS. The simulation results for such an objective are shown in Fig. 3(a)–(d). It can be seen that both the controllers exert excellent control actions and no offsets are observed.

3. Controlling the system at the unique USS responsible for chaotic motion

In this objective, the controllers, with the system beginning at an arbitrary point in the phase space, are required to stabilize the chaotic trajectory (shown in Fig. 1(c) and (d)) exactly at the corresponding unique USS. For simulating controller actions, the parameter set III is used with the setpoint: $x_1^{set}=0.0819$, $x_2^{set}=0.1391$ and $x_3^{set}=3.7627$. Fig. 4(a) and (b), and (c) and (d) show the x_3 and u_t time profiles in the presence of actions delivered by controllers 1 and 2, respectively. These plots indicate well that both the controllers fulfill the control objective of stabilizing chaotic motion at the USS without allowing any offset.

4. Controlling the system at USS responsible for chaotic motion in the presence of stochastic and deterministic load disturbances

The ability of the proposed controllers to impart the desired control action in the presence of stochastic load disturbances was tested by incorporating random noise at every integration step in the time evolution equation for reactor temperature (Eq. (13)). Thus the load disturbance term d_3 assumes random values dictated by the Gaussian distribution having a mean and standard deviation of 0 and 0.25, respectively. The process and controller outputs in the presence of such random

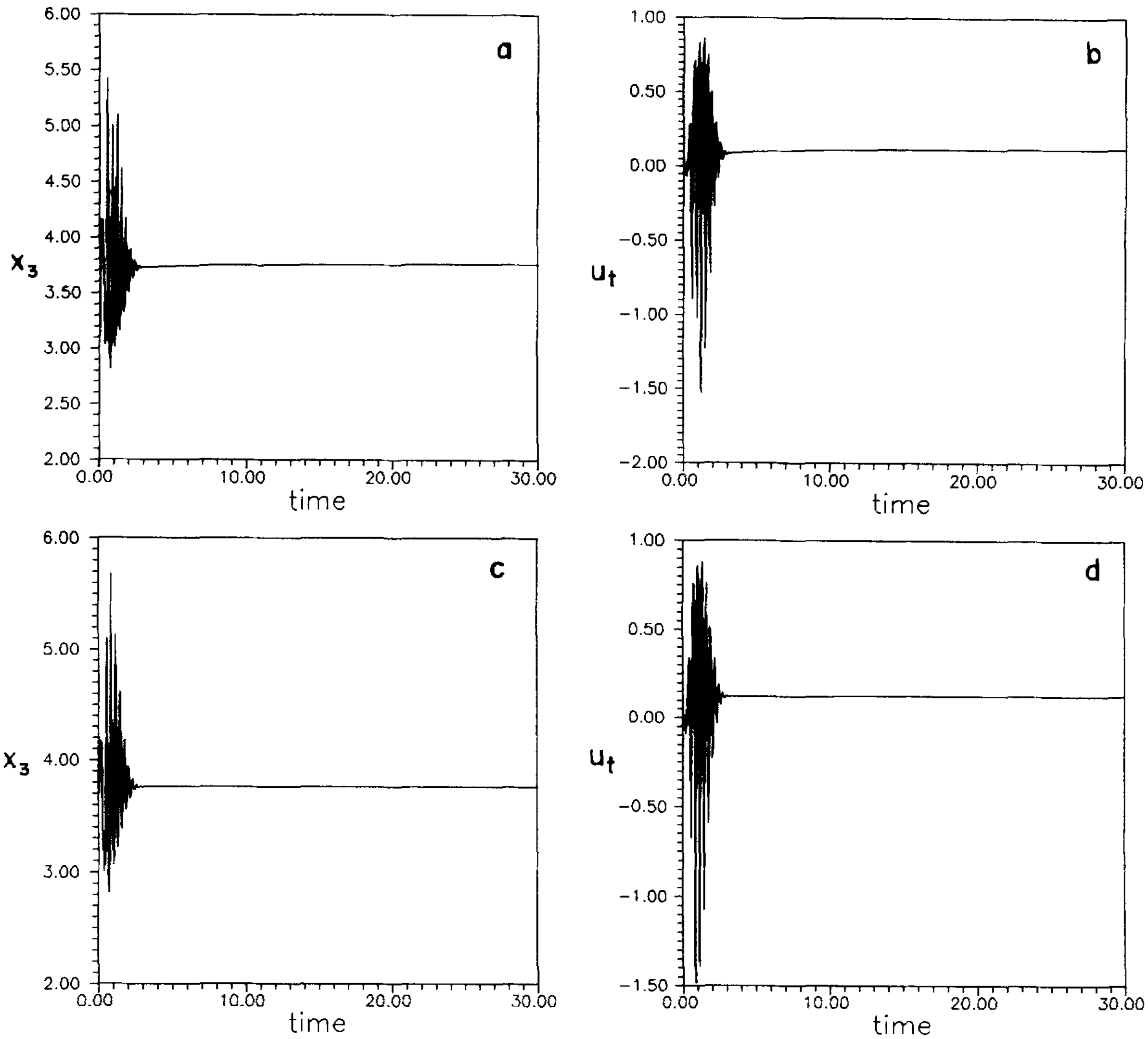

Fig. 6. Same as Fig. 4 but in the presence of fixed (deterministic) load disturbance of unit magnitude ($d_3=0$).

disturbances are shown in Fig. 5. In addition, the results when a constant (deterministic) load disturbance of unit magnitude is added ($d_3=1$) are depicted in Fig. 6(a)–(d). As can be noted from Figs. 5 and 6, the controllers deliver excellent action in the presence of either type of load disturbances.

5. Case study 2

Here, we consider the kinetic model satisfying the mass-action law studied by Gaspard and Nicolis [22] and Nicolis [23]. The model equations which in certain parameter space are known to exhibit homoclinic chaos are

$$\frac{\mathrm{d}x}{\mathrm{d}t}=x(\hat{d}x-\hat{f}y-z+g)+d_1 \tag{16}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=y(x+\hat{s}z-l)+d_2 \tag{17}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t}=\left(\frac{1}{\hat{\epsilon}}\right)(x-az^3+\hat{b}z^2-cz)+d_3 \tag{18}$$

These equations for the parameter values shown in Table 1 and in the absence of load disturbances ($d_1=d_2=d_3=0$) possess three steady states all of which have been found to be unstable. The resultant chaotic behavior is depicted in Fig. 7. Computer simulations involving control are conducted analogous to that described for Case study 1 wherein the control variable u_t now signifies the deviation in the model parameter g. Thus, in the presence of control action, Eq. (16) is modified to

$$\frac{\mathrm{d}x}{\mathrm{d}t}=x(\hat{d}x-\hat{f}y-z+g+u_t)+d_1 \tag{19}$$

The performance of controllers 1 and 2 in controlling the chaotic dynamics exhibited by Eqs. (16)–(18) has been studied by setting up two relevant objectives and the results of such simulations are described below.

1. Controlling the system trajectory at a USS in the chaotic parameter regime

This controller task is the same as objective (3) in Case study 1. The process setpoint is chosen as the first USS:

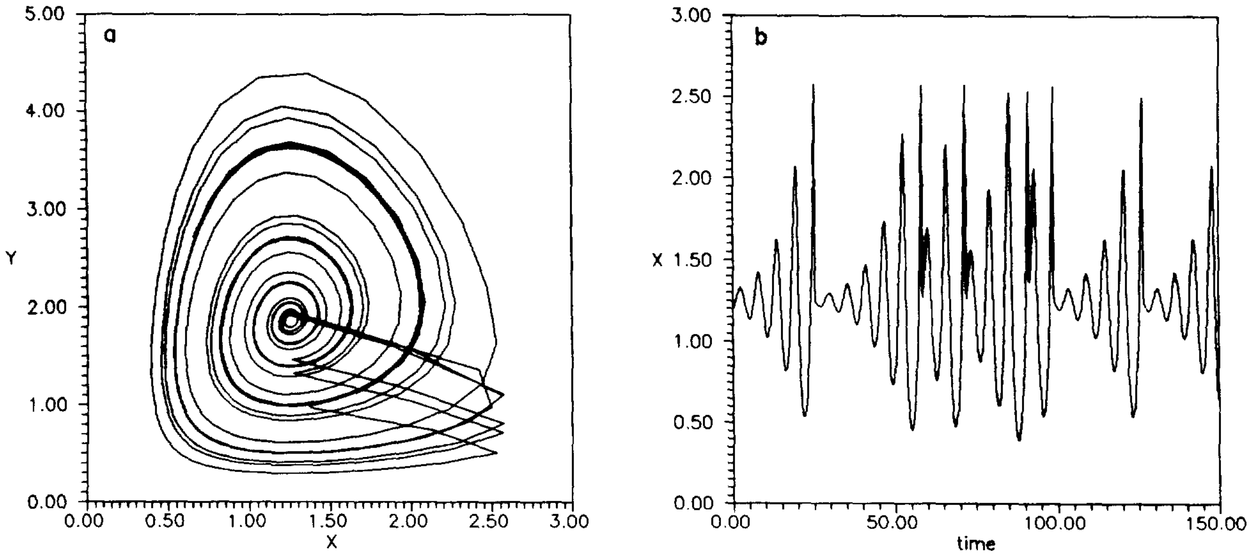

Fig. 7. The phase plane plot (a) showing the chaotic attractor in the x–y plane of the system analyzed as case study 2. (b) depicts the x time profile corresponding to (a).

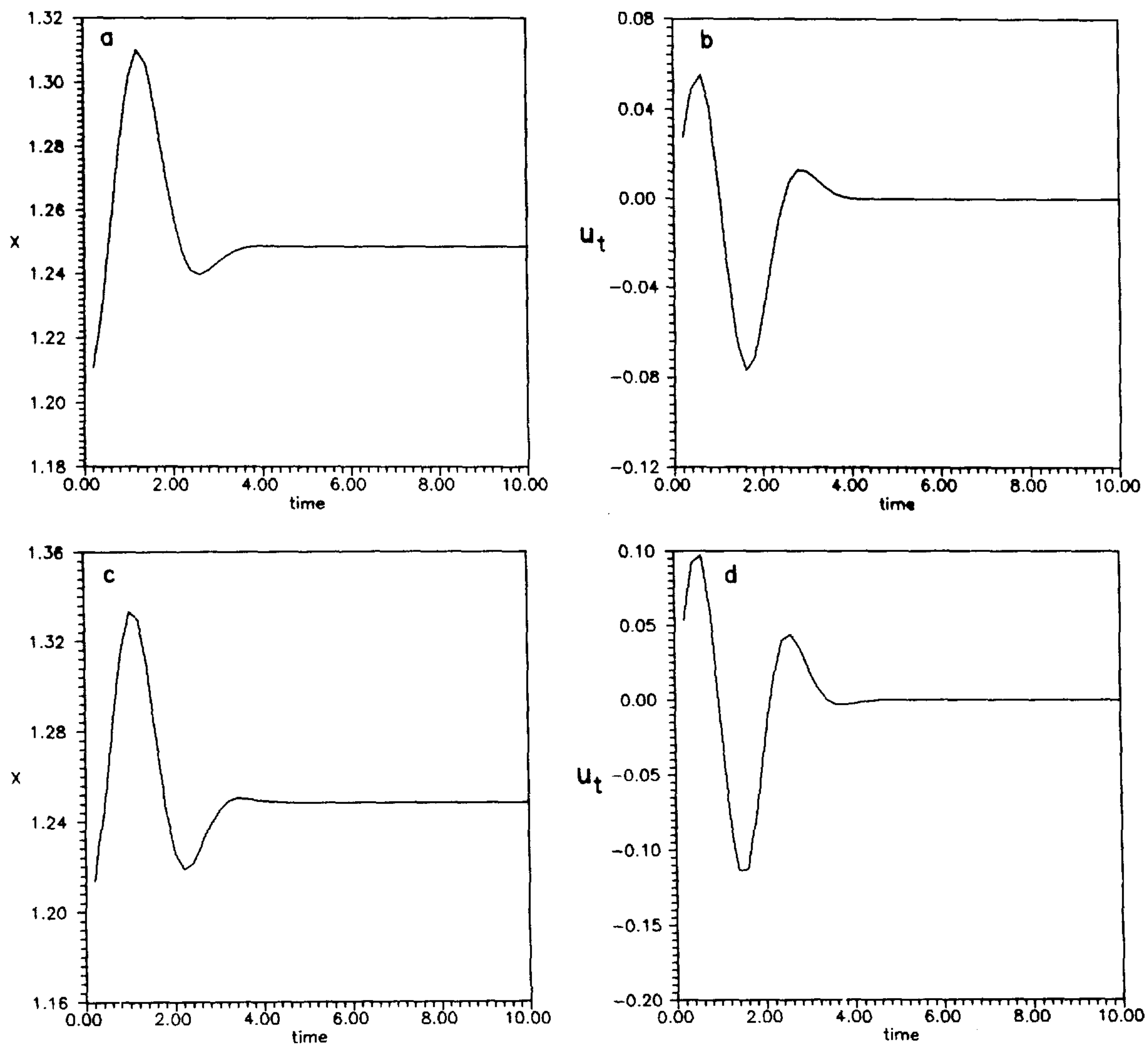

Fig. 8. Plots of process outputs (a, c) and controller outputs (b, d) obtained with controller-1 (a, b) and controller-2 (c, d). The plots pertain to objective 1 of case study 2 where the setpoint is an USS in the multiplicity region. (a) and (b) show the time profiles of x and controller-1 output u_t. (c) and (d) depict the time profiles of the same variables but for controller-2.

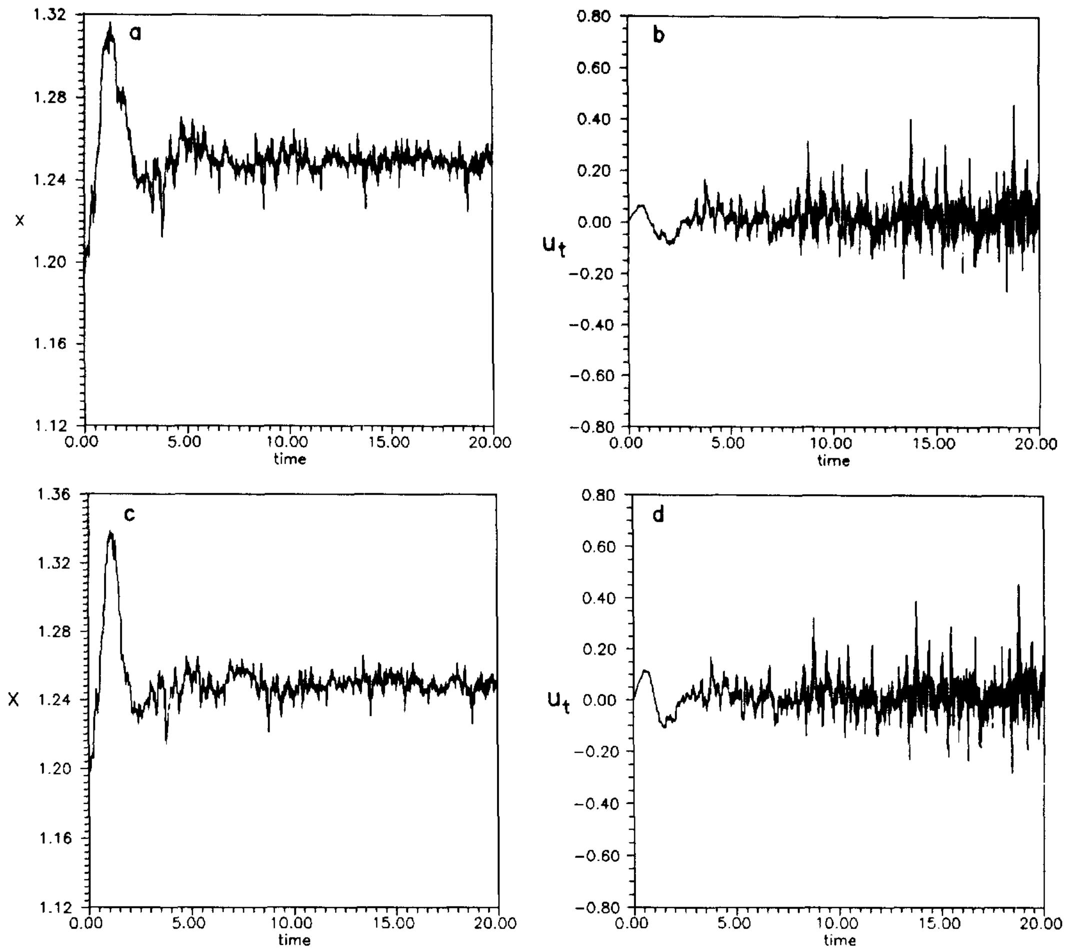

Fig. 9. The description of trajectories is the same as for Fig. 8 except that the process contains stochastic load disturbance (d_1) obeying Gaussian distribution with mean and standard deviation equal to 0 and 0.15 respectively.

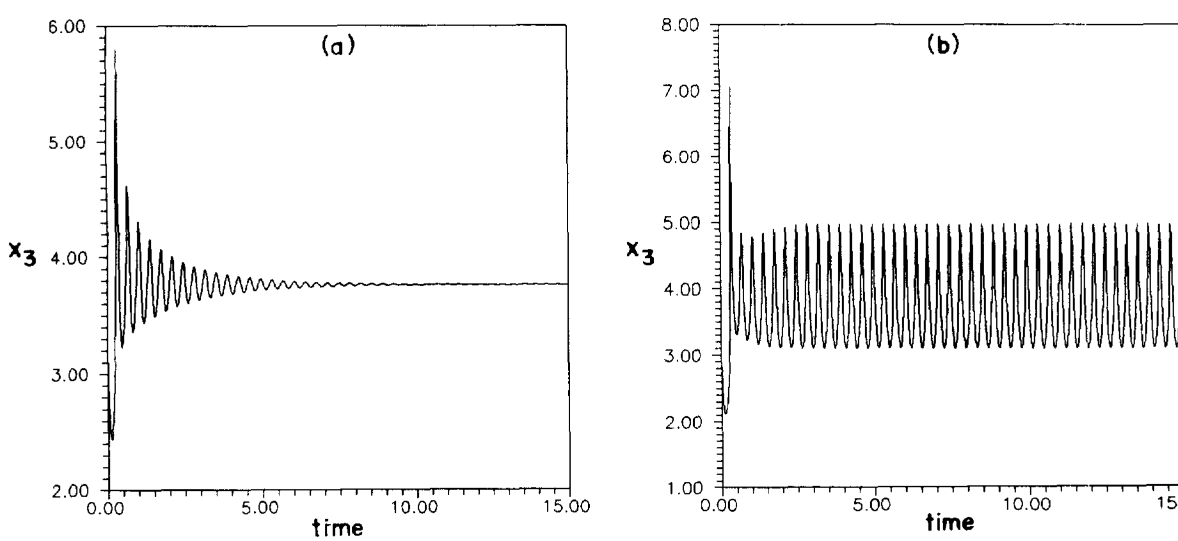

Fig. 10. Plots of process output, x_3, as a function of time obtained with PID control. (a) corresponds to $K_c = 0.15$, $\tau_I = 0.2$, $\tau_D = 0.2$ and (b) refers to $K_c = 0.11$, $\tau_I = 0.2$, $\tau_D = 0.2$.

$x_s = 1.24855$, $y_s = 1.8705$, and $z_s = 0.3015$. The results of the executed control actions when the system begins at an arbitrary point are shown in Fig. 8(a)–(d) which clearly suggest that the control objective is fulfilled satisfactorily. That is, the controllers do stabilize the system exactly at the USS and no offsets are observed.

2. Controlling the system at a USS in the chaotic parameter regime in the presence of stochastic load disturbance

To test the robustness of the proposed controllers, Gaussian random noise with mean = 0 and standard deviation = 0.15 was added to Eq. (16). Thus, the d_1 term assumes random values; its update takes place at every 0.001 time units (integration step). As can be seen from Fig. 9(a) and (c), both the controllers again exert excellent control action in the presence of continuous random load disturbances.

6. Comparison with other controllers

For comparison purposes, we conducted simulations involving PID control and the fixed gain variants of controllers 1 and 2. The general form of the conventional PID controller is

$$u_t = \overline{u_t} + K_c\left(e + \frac{1}{\tau_I}\int e\,dt + \tau_D\frac{de}{dt}\right) \qquad (20)$$

where K_c denotes the controller gain and $\bar{u}_t$ refers to the controller output when the setpoint error e is zero. The integral and derivative time constants are represented by τ_I and τ_D, respectively.

The results of simulation with PID control for Case study I are shown in Fig. 10. While PID control is shown to be

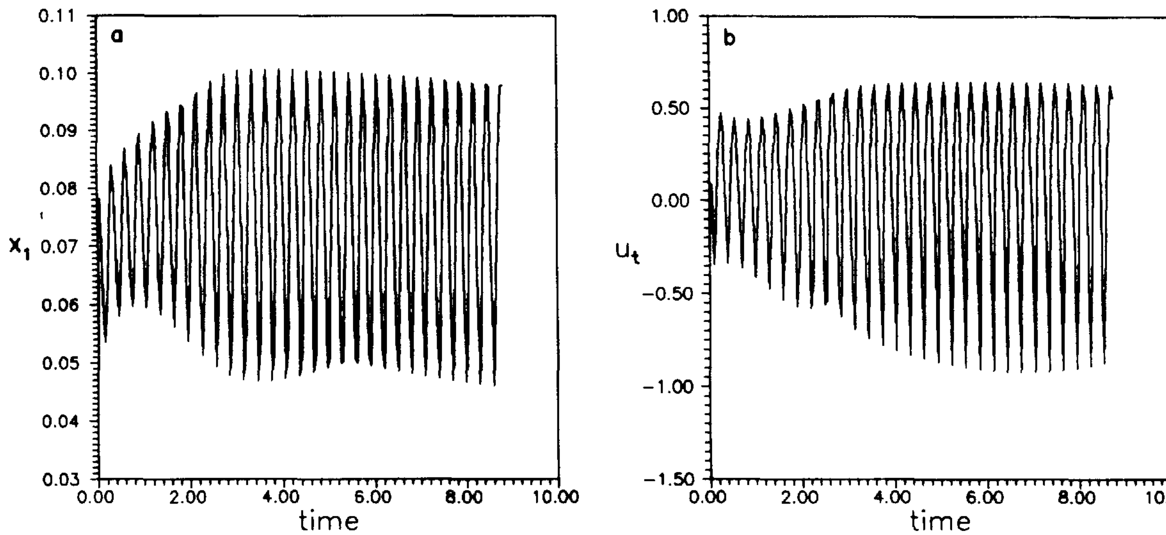

Fig. 11. Plots of process output (a) and controller output (b) obtained with fixed-gain controller-2. The plots pertain to the control objective 3 for the nonlinear CSTR process where the setpoint is the unique USS responsible for chaotic motion.

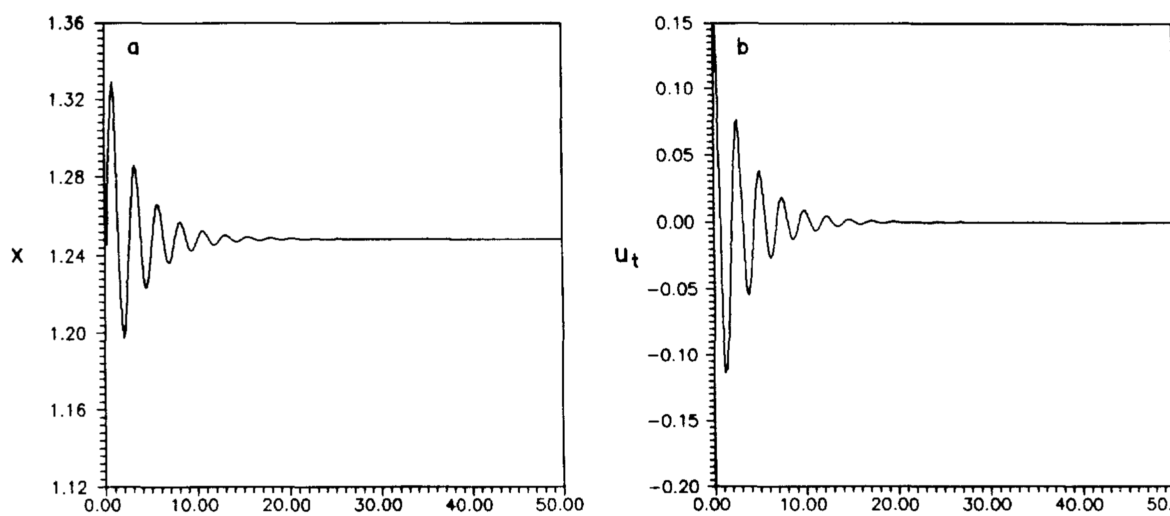

Fig. 12. Process (a) and controller (b) outputs (Case study 2; objective 1) when the gains of the proportional and derivative terms (Eq. 4) are kept fixed.

capable of restoring the chaotic system to the desired unstable steady-state as shown in Fig. 10(a), the system performance is extremely sensitive to the proportional gain. For example, for a small change (from 0.15 to 0.11) in the proportional gain, the ability of the control system to achieve the desired objective is lost, as shown in Fig. 10(b). Furthermore, to find the suitable controller parameters has been difficult.

We also evaluated the performance of the proposed controllers when the respective gains of the proportional, derivative, and integral terms appearing on the right-hand sides of Eqs. (4) and (7) were held constant. The motivation behind such an exercise was to check the efficacy of proposed controllers in the absence of gain-adaptation. In these simulations, the proportional, derivative, and integral terms of the proposed controllers were studied separately and in combination with each other. One of the possible fixed-gain feedback controllers with proportional-only terms on its rhs has the form equivalent to Eq. (1). It was found that none of the controllers with their gains fixed can satisfy any of the four control goals that are set for the system representing the nonisothermal CSTR. For example, the fixed gain equivalent of controller-2 when employed towards the objective (3) gives rise to x_1 and u_t profiles shown in Fig. 11. It can be seen that the controller output oscillates resulting in overall oscillatory system behavior. However, for the reaction model studied in Case study 2, it was found that only a particular combination, the one comprising the proportional and derivative terms, with their gains fixed, is capable of stabilizing the system trajectory at the USS represented by $x_s = 1.24855$, $y_s = 1.8705$ and $z_s = 0.3015$. The results of this simulation are portrayed in Fig. 12. It is possible to compare the control action delivered by the fixed gain controller (Fig. 12(a) and (b)) with those effected by the controllers 1 and 2 (Fig. 8(a)–(d)). It is noticed easily that the actions of controllers 1 and 2 are much smoother and the set point is reached earlier.

7. Conclusion

In this paper, the modified forms of the feedback control mechanism introduced as Eqs. (4) and (7) have been employed to control successfully the continuous nonlinear dynamical systems exactly at their unstable steady states. The performance of these controllers has been evaluated by considering two reaction systems exhibiting multi-stationarity, oscillations and even chaos. Simulation results clearly indicate that the proposed controllers provide a very good alternative for controlling the unstable dynamics that arise due to unstable steady states. The proposed gain-adapting controllers are capable of fast response and fulfill the control objectives even in the presence of deterministic or stochastic load disturbances. Although the proposed controllers employ a simple time-dependent linear variation of the controller gain, in principle, it is possible to formulate other gain-adapting strategies.

The characteristic behavior of the proposed controllers as can be perceived from Eqs. (4) and (7) is that the manipulated variable u_t, when the control action is switched on moves rapidly under the influence of large magnitudes of e and de/dt. As time progresses, the system evolves towards the setpoint, and as a result the setpoint error term e and, consequently, de/dt tend to zero. At this stage t is much larger compared to e and its time-derivative (de/dt), which again results in the faster system movement towards the setpoint.

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